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ON THE HYPERSONIC FLOW PAST A LIFT AIRFOIL

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The hypersonic flow past a wing profile subjected to lift is considered. Effects of viscosity anf thermal conductivity in the region of flow outside the trail are neglected. An analogy is formulated which makes it possible to determine the velocity field by solving the problem of "directional" explosion in which not only energy but, also, momentum are imparted to gas. Motion within the viscous trail is specified by two terms of the asymptotic expansion of the solution of Navier-Stokes equations.

1. The outer region. Let us consider the hypersonic flow past a wing of infinite span. We denote the density of gas in the oncoming stream by ρ_∞ and by v_∞ its velocity in the direction of the x-axis of the Cartesian system of coordinates xy. We assume that upstream of the bow shock wave shown in Fig. 1 the pressure $p_\infty = 0$ and, consequently, the Mach number $M_\infty = \infty$. The gas is assumed to be perfect, i.e. to conform to the equation of state for such gas (the Clapeyron law) and that both specific heats c_p and c_v are constant. For simplicity we assume that the dependence of the coefficients of viscosity λ and thermal conductivity k on specific enthalpy is linear: $\lambda = \lambda_0 w$ and $k = k_0 w$. In the following analysis it is expedient to specify both the independent variables and the unknown functions in dimensionless form, taking ρ_∞, v_∞ and λ_0 as the fundamental units of reference.

At some distance from the body the shape of the shock wave $y_2(x)$ is primarily determined by wave drag, while its inner structure depends on dissipative processes taking place in the medium. For a gas with $\lambda = k = 0$ an analogy was formulated in [1-4] in which the unsteady motion in a space with the number of measurements reduced by one simulates a hypersonic stream. With the use of that analogy it is possible to determine the velocity field associated with the drag of a body on the basis of the solution of the problem of a strong explosion [5-9]. The law established by Sedov [10, 11] for the propagation of explosion waves when applied to hypersonic flows states that $y_2 \sim x^{2/3}$. It follows from this that for $x \to \infty$ the asymptotic expansion for the transverse coordinate of a compression shock may be written as

$$y_2 = (bx)^{2/3} \left(\pm 1 + b_m x^{-2m/3} + \ldots\right) \tag{1.1}$$

where constant b_m and exponent m are to be determined from the condition that lift F_y must be independent of the choice of reference planes at $x = \text{const}$ which are used for computing the component of momentum of gas passing along the y-axis through these up- and downstream of the body. The positive and negative signs of the first term in parentheses relate to the upper and lower half-planes, respectively.

We denote the stream function by ψ. Solution of the problem of a strong explosion necessitates the introduction of the self-similar combination

$$\eta = \psi (bx)^{-2/3} \tag{1.2}$$

as one of the independent variables.

Let us denote by v_x and v_y the projections of the velocity vector on the x- and y-axes, respectively, set $\varkappa = c_p / c_v$, and seek the expansion for the parameters of gas in the form

$$\begin{aligned}
v_x &= 1 - \frac{8}{9(\varkappa+1)}\left(\frac{b^2}{x}\right)^{2/3}\left[v_{x11}(\eta) + b_m x^{-2m/3} v_{x12}(\eta) + \ldots\right] \qquad (1.3)\\
v_y &= \frac{4}{3(\varkappa+1)}\left(\frac{b^2}{x}\right)^{1/3}\left[v_{y11}(\eta) + b_m x^{-2m/3} v_{y12}(\eta) + \ldots\right]\\
\rho &= \frac{\varkappa+1}{\varkappa-1}\left[\rho_{11}(\eta) + b_m x^{-2m/3}\rho_{12}(\eta) + \ldots\right]\\
p &= \frac{8}{9(\varkappa+1)}\left(\frac{b^2}{x}\right)^{2/3}\left[p_{11}(\eta) + b_m x^{-2m/3} p_{12}(\eta) + \ldots\right]\\
w &= \frac{8\varkappa}{9(\varkappa+1)^2}\left(\frac{b^2}{x}\right)^{-2/3}\left[w_{11}(\eta) + b_m x^{-2m/3} w_{12}(\eta) + \ldots\right]\\
y &= (bx)^{2/3}\left[y_{11}(\eta) + b_m x^{-2m/3} y_{12}(\eta) + \ldots\right]
\end{aligned}$$

Let us consider the initial data which are to be satisfied by the first and second approximation functions. It was shown by Sychev [12] that a strong shock wave at $M_\infty = \infty$ has a boundary at its front. The derivatives of parameters of a viscous heat-conducting gas passing through that boundary become discontinuous. According to [13, 14] it is possible to specify the discontinuity line of derivatives by formula (1.1) and obtain the solution of the Navier-Stokes equations which define the inner structure of a compression shock. At a reasonably great distance from the boundary separating the latter from the oncoming stream the principal part of the solution yields initial values of functions (1.3). The latter can, thus, be used for determining the velocity field in the region adjacent to the shock wave smoothed by viscosity and thermal conductivity. It will be readily seen that the initial data which follow from the Hugoniot conditions at the front of a strong discontinuity in a perfect gas are also valid for functions $v_{x11}, \ldots, y_{11}$. Moreover, if in formula (1.1) the exponent $m < {}^3/_2$, the initial data for functions $v_{x12}, \ldots, y_{12}$ are exactly the same as those following from Hugoniot's formulas.

We draw the reference planes normal to the direction of the oncoming stream, one up- the other downstream of the body at distance x (Fig. 1). To derive the expression for lift F_y we calculate the component of the momentum of gas passing through these planes and projected on the y-axis. We have

$$F_y = -\frac{8}{3(\varkappa+1)} b^{4/3} b_m B x^{-(2m-1)/3}, \qquad B = 1 + \int_0^1 v_{y12}(\eta)\, d\eta \tag{1.4}$$

By setting $m = {}^1/_2$, we eliminate in the last formula the x-coordinate. The above reasoning obviously implies that for $m = {}^1/_2$ it is immaterial for the derivation of initial values of unknown functions whether the discontinuity line of gas parameter derivatives, which bounds the diffused compression shock, is specified by equality (1.1) or by the simple shock front with Hugoniot's conditions at it.

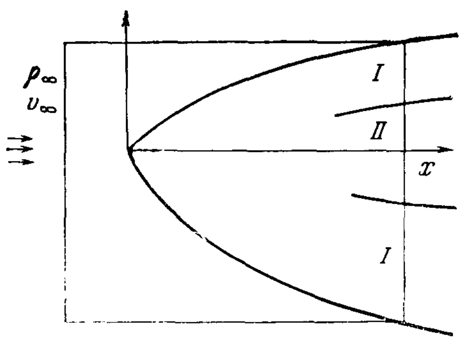

Fig. 1

Let us now turn to the Navier-Stokes equations. Denoting the Prandtl number by N_{Pr} and using Mieses' variables these equations are of the form

$$\rho v_x \frac{\partial v_x}{\partial x} + \frac{\partial p}{\partial x} - \rho v_y \frac{\partial p}{\partial \psi} = \left(\frac{\partial}{\partial x} - \rho v_y \frac{\partial}{\partial \psi}\right)\left[w\left(\frac{4}{3}\frac{\partial v_x}{\partial x} - \frac{4}{3}\rho v_y \frac{\partial v_x}{\partial \psi} - \frac{2}{3}\rho v_x \frac{\partial v_y}{\partial \psi}\right)\right] + \rho v_x \frac{\partial}{\partial \psi}\left[w\left(\rho v_x \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial x} - \rho v_y \frac{\partial v_y}{\partial \psi}\right)\right] \tag{1.5}$$

$$\rho v_x \frac{\partial v_y}{\partial x} + \rho v_x \frac{\partial p}{\partial \psi} = \rho v_x \frac{\partial}{\partial \psi}\left[w\left(\frac{4}{3}\rho v_x \frac{\partial v_y}{\partial \psi} - \frac{2}{3}\frac{\partial v_x}{\partial x} - \frac{2}{3}\rho v_y \frac{\partial v_x}{\partial \psi}\right)\right] + \left(\frac{\partial}{\partial x} - \rho v_y \frac{\partial}{\partial \psi}\right)\left[w\left(\rho v_x \frac{\partial v_x}{\partial \psi} - \frac{\partial v_x}{\partial x} - \rho v_y \frac{\partial v_y}{\partial \psi}\right)\right]$$

$$\rho v_x \frac{\partial w}{\partial x} - v_x \frac{\partial p}{\partial x} = \frac{1}{N_{Pr}}\left\{\left(\frac{\partial}{\partial x} - \rho v_y \frac{\partial}{\partial \psi}\right)\left[w\left(\frac{dw}{\partial x} - \rho v_y \frac{\partial w}{\partial \psi}\right)\right] + \rho v_x \frac{\partial}{\partial \psi}\left(\rho w v_x \frac{\partial w}{\partial \psi}\right)\right\} + w\left\{2\left[\left(\frac{\partial v_x}{\partial x} - \rho v_x \frac{\partial v_x}{\partial \psi}\right)^2 + \left(\rho v_x \frac{\partial v_y}{\partial \psi}\right)^2\right] + \left(\rho v_y \frac{\partial v_x}{\partial \psi} + \frac{\partial v_y}{\partial x} - \rho v_y \frac{\partial v_y}{\partial \psi}\right)^2 - \frac{2}{3}\left(\frac{\partial v_x}{\partial x} - \rho v_y \frac{\partial v_x}{\partial \psi} + \rho v_x \frac{\partial v_x}{\partial \psi}\right)^2\right\}$$

$$\rho v_x \frac{\partial y}{\partial \psi} = 1, \qquad v_x \frac{\partial y}{\partial x} = v_y, \qquad p = \frac{\varkappa - 1}{\varkappa}\rho w$$

Substituting expansions (1.3) into the Navier-Stokes equations, we obtain two systems of ordinary differential equations. The nonlinear system of the first approximation yields the solution of the problem of strong explosion, whose closed form was given by Sedov [10, 11]. The system of the second approximation

$$v_{x12} = \frac{1}{\varkappa+1}\left[2v_{y11}v_{y12} + \varkappa\left(\frac{p_{12}}{\rho_{11}} - \frac{p_{11}\rho_{12}}{\rho_{11}^2}\right)\right] \tag{1.6}$$

$$\frac{dp_{12}}{d\eta} - \eta\frac{dv_{y12}}{d\eta} = v_{y12}, \qquad p_{12} - \varkappa\frac{p_{11}\rho_{12}}{\rho_{11}} = 2\eta^{-3/2}\rho_{11}^{\varkappa}$$

$$\rho_{11}\frac{dy_{12}}{d\eta} = -\frac{dy_{11}}{d\eta}\rho_{12}, \qquad \eta\frac{dy_{12}}{d\eta} = \frac{1}{2}y_{12} - \frac{2}{\varkappa+1}v_{y12}$$

$$p_{12} = \rho_{11}w_{12} + w_{11}\rho_{12}$$

is linear.

It is immediately seen that this system is independent of terms related to viscosity and thermal conductivity which appear in Eqs. (1.5) in the case of a real medium. Thus the perturbation field structure in the external region bounded upstream by the shock wave front is in the first and second approximation independent of dissipative effects. We may also note that the first of Eqs. (1.6) stands out from the others in that it is used for finding v_{x12} after functions $v_{y12}, \ldots, y_{12}$ have been determined. The latter satisfy the system of equations which arises in the investigation of the second approximation in the theory of one-dimensional nonstationary motions. This implies that within the specified accuracy the perturbation field outside the trail can be constructed by using the principle of equivalence [1 – 4] according to which stream parameters in any plane x = const can be determined independently of the values of its parameters in other planes.

Equations (1.6) must be integrated from points $\eta = \pm 1$, where

$$v_{x12} = \pm\frac{4-\varkappa}{\varkappa+1}, \qquad v_{y12} = \frac{2-\varkappa}{\varkappa+1}, \qquad \rho_{12} = \mp\frac{3}{\varkappa+1}$$

$$p_{12} = \pm\frac{2-\varkappa}{\varkappa+1}, \qquad w_{12} = \frac{5-\varkappa}{\varkappa+1}, \qquad y_{12} = \frac{2}{\varkappa+1}$$

respectively.

2. Analogy with directional explosion. With allowance for initial data the second of Eqs. (1.6) yields

$$p_{12} = \eta v_{y12} \tag{2.1}$$

In the theory of one-dimensional nonstationary motion time t represents the x-coordinate, the quantity

$$\xi = y\,(bt)^{-2/3} = y_{11}(\eta) + b_{1/2}\,t^{-1/3}\,y_{12}(\eta) + \ldots$$

is taken as the self-similar combination instead of (1.2), and the expansion of expressions for velocity, density, and pressure are of the form

$$v = \frac{4}{3(\varkappa+1)}\left(\frac{b^2}{t}\right)^{1/3}\left[f(\xi) + b_{1/2}t^{-1/3}f_{1/2}(\xi) + \ldots\right] \tag{2.2}$$

$$\rho = \frac{\varkappa+1}{\varkappa-1}\left[g(\xi) + b_{1/2}t^{-1/3}g_{1/2}(\xi) + \ldots\right]$$

$$p = \frac{8}{9(\varkappa+1)}\left(\frac{b^2}{t}\right)^{2/3}\left[h(\xi) + b_{1/2}t^{-1/3}h_{1/2}(\xi) + \ldots\right]$$

It is shown in [15] that variation functions are of the form of integral

$$\xi\,(gf_{1/2}+fg_{1/2})-\frac{1}{\varkappa+1}\,[4fgf_{1/2}+2f^2g_{1/2}+(\varkappa-1)\,h_{1/2}]=0 \qquad (2.3)$$

which conforms to the Hugoniot conditions for a strong shock front. Let us reduce formula (2.1) to the form (2.3). To do this, we first of all establish the expression for the increment of the transverse coordinate y_{12}. Using the relationship

$$\frac{dy_{11}}{d\eta}=\frac{\varkappa-1}{\varkappa+1}\,\frac{1}{\rho_{11}}$$

between functions of the first approximation and of the fourth and fifth of Eqs. (1.6), we obtain

$$y_{12}=\frac{1}{\varkappa+1}\left[4v_{y12}-2\,(\varkappa-1)\,\eta\,\frac{\rho_{12}}{\rho_{11}^2}\right] \qquad (2.4)$$

Comparison of expansions (1.3) and (2.2) yields

$$v_{y_{12}}=f_{1/2}+y_{12}\frac{df}{d\xi}, \qquad \rho_{12}=g_{1/2}+y_{12}\frac{dg}{d\xi}, \qquad p_{12}=h_{1/2}+y_{12}\frac{dh}{d\xi}$$

Substituting into these the expression (2.4) for y_{12}, we derive two equations which determine v_{y12} and ρ_{12}. By solving the obtained equations with allowance for $\eta=\rho_{11}^{\varkappa}/p_{11}=g^{\varkappa}/h$, we obtain solutions

$$v_{y_{12}}=\left(1+\frac{4}{\varkappa+1}\frac{df}{d\xi}\right)f_{1/2}+\frac{4}{\varkappa+1}\,\frac{1}{g}\left(f-\frac{\varkappa+1}{2}\,\xi\right)\frac{df}{d\xi}\,g_{1/2}$$

$$\rho_{12}=\frac{4}{\varkappa+1}\frac{dg}{d\xi}\,f_{1/2}+\left(1-\frac{4}{\varkappa+1}\frac{df}{d\xi}\right)g_{1/2}$$

We now readily find expressions for pressure increments

$$p_{12}=\frac{4}{\varkappa+1}\frac{dh}{d\xi}\,f_{1/2}+\frac{4}{\varkappa+1}\,\frac{1}{g}\left(f-\frac{\varkappa+1}{2}\,\xi\right)\frac{dh}{d\xi}\,g_{1/2}+h_{1/2}$$

We substitute the derived expressions for functions v_{y12} and p_{12} into formula (2.1). Simple transformations show that the latter is a different form of integral (2.3). An important qualitative conclusion follows from this. As shown in [15], integral (2.3) exists only when the expression for the momentum of the medium in the perturbed region contains a term indeoendent of time. In relation to the considered problem this means that not only energy, but also momentum is imparted to the gas by the explosion. In every plane $x=$ const parameters of a hypersonic flow around a wing are the same as those of an explosion wave whose propagation is accompanied by momentum transfer along the y-axis.

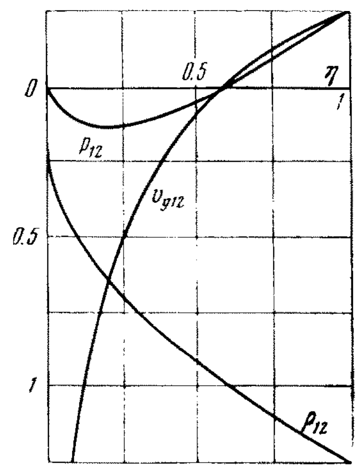

Fig. 2

Formula (2.1) makes it possible to derive a first order differential equation for function v_{y12} and obtain its solution in quadratures which, owing to its unwieldiness, is not presented here. Integration of the equation for v_{y12} by numerical methods is much more convenient. The behavior of functions v_{y12}, ρ_{12} and p_{12} is shown in Fig. 2.

The asymptotics of the first approximation function for $\eta\to\pm 0$ is known from the solution of the problem of a strong explosion

$$v_{x11} = \frac{\varkappa}{\varkappa+1} h_0^{(\varkappa-1)/\varkappa} |\eta|^{-1/\varkappa} + \ldots, \quad v_{y11} = \pm\frac{1}{2} h_0^{-1/\varkappa} |\eta|^{(\varkappa-1)/\varkappa} + \ldots$$

$$\rho_{11} = h_0^{1/\varkappa} |\eta|^{1/\varkappa} + \ldots, \quad p_{11} = h_0 + \ldots, \quad w_{11} = h_0^{(\varkappa-1)/\varkappa} |\eta|^{-1/\varkappa} + \ldots$$

$$y_{11} = \pm\frac{\varkappa}{\varkappa+1} h_0^{-1/\varkappa} |\eta|^{(\varkappa-1)/\varkappa} + \ldots, \qquad h_0 = \frac{9}{8}(\varkappa+1) k_2$$

Coefficients k_2 can be found in Sedov's monograph [16]. Second approximation functions for $\eta \to \pm 0$ are defined by asymptotic formulas

$$v_{x12} = \pm\frac{2}{\varkappa+1} h_0^{(\varkappa-1)/\varkappa} |\eta|^{-(2+\varkappa)/2\varkappa} + \ldots \tag{2.5}$$

$$v_{y12} = -\frac{2(\varkappa-1)}{\varkappa(2-\varkappa)} \eta_0^{-1/\varkappa} |h|^{-(2-\varkappa)/2\varkappa} + \ldots$$

$$\rho_{12} = \mp\frac{2}{\varkappa} h_0^{1/\varkappa} |\eta|^{(2-\varkappa)/2\varkappa} + \ldots, \qquad p_{12} = \mp\frac{2(\varkappa+1)}{\varkappa(2-\varkappa)} h_0^{-1/\varkappa} |\eta|^{(3\varkappa-2)/2\varkappa} + \ldots$$

$$w_{12} = \pm\frac{2}{\varkappa} h_0^{(\varkappa-1)/\varkappa} |\eta|^{-(2+\varkappa)/2\varkappa} + \ldots$$

$$y_{12} = -\frac{4(\varkappa-1)}{(\varkappa+1)(2-\varkappa)} h_0^{-1/\varkappa} |\eta|^{-(2-\varkappa)/2\varkappa} + \ldots$$

In these expansions the pressure increment $p_{12} = 0$ when $\eta = 0$.

3. The laminar trail. In the vortex trail region it is no longer possible to neglect viscosity and thermal conductivity. Dissipative effects play a major part in that region. Sychev, who had compared the relative magnitude of convective terms and terms affected by heat transfer in the Navier-Stokes equations [17], came to the conculsion that the analysis of the trail of (1.2) necessitates the introduction of the new self-similar variable

$$\zeta = \psi\,(bx)^{-1/6} = \eta x^{1/2} \tag{3.1}$$

Expansions for gas parameters are of the form

$$v_x = 1 - \frac{8\varkappa}{9(\varkappa+1)^2} b^{4/3} h_0^{(\varkappa-1)/\varkappa} x^{(3-4\varkappa)/6\varkappa} \left[v_{x21}(\zeta) + \frac{2}{\varkappa} b_{1/2} x^{-1/12} v_{x22}(\zeta) + \ldots\right. \tag{3.2}$$

$$v_y = \frac{2}{3(\varkappa+1)} b^{2/3} h_0^{-1/\varkappa} x^{-(5\varkappa-3)/6\varkappa} \left[v_{y21}(\zeta) - \frac{4(\varkappa-1)}{\varkappa(2-\varkappa)} b_{1/2} x^{-1/12} v_{y22}(\zeta) + \ldots\right]$$

$$\rho = \frac{\varkappa+1}{\varkappa-1} h_0^{1/\varkappa} x^{-1/2\varkappa} \left[\rho_{21}(\zeta) - \frac{2}{\varkappa} b_{1/2} x^{-1/12} \rho_{22}(\zeta) + \ldots\right]$$

$$p = \frac{8}{9(\varkappa+1)} b^{4/3} h_0 x^{-2/3} [p_{21}(\zeta) + \ldots]$$

$$w = \frac{8\varkappa}{9(\varkappa+1)^2} b^{4/3} h_0^{(\varkappa-1)/\varkappa} x^{(3-4\varkappa)/6\varkappa} \left[w_{21}(\zeta) + \frac{2}{\varkappa} b_{1/2} x^{-1/12} w_{22}(\zeta) + \ldots\right]$$

$$y = \frac{\varkappa}{\varkappa+1} b^{2/3} h_0^{-1/\varkappa} x^{(3+\varkappa)/6\varkappa} \left[y_{21}(\zeta) - \frac{4(\varkappa-1)}{\varkappa(2-\varkappa)} b_{1/2} x^{-1/12} y_{22}(\zeta) + \ldots\right]$$

Only one term is retained in the formula for pressure, since the correction to p_{21} is of the order of $x^{-(2\varkappa-1)/2\varkappa} \ll x^{-1/12}$ when $x \to \infty$.

Let us assume that formulas (1.3) and (3.2) specify expansions for the outer and inner regions, respectively. Matching of these two expansions is based on the existence of a region where they overlap [18, 19]. This provides the boundary conditions which must be satisfied by the unknown functions for $|\zeta| \to \infty$. The subsequent analysis requires not only knowledge of the boundary conditions, but also a correct estimate of the usually neglected remainder terms. Formulas (1.3) are insufficient for establishing the form of the latter, and terms of higher order of smallness than those so far considered must be added to the right-hand sides of these formulas. We omit, for brevity, all computations and present only the final results for the first approximation functions. For $\zeta \to \pm\infty$ we have

$$v_{x21} \to |\zeta|^{-1/\varkappa} + O(|\zeta|^{-(2\varkappa+1)/\varkappa}), \quad y_{y21} \to \pm|\zeta|^{(\varkappa-1)/\varkappa} + O(|\zeta|^{-(\varkappa+1)/\varkappa}) \tag{3.3}$$
$$\rho_{21} \to |\zeta|^{1/\varkappa} + O(|\zeta|^{(1-2\varkappa)/\varkappa}), \qquad p_{21} \to 1$$
$$w_{21} \to |\zeta|^{-1/\varkappa} + O(|\zeta|^{-(2\varkappa+1)/\varkappa}), \quad y_{21} \to \pm|\zeta|^{(\varkappa-1)/\varkappa} + O(|\zeta|^{-(\varkappa+1)/\varkappa})$$

Boundary conditions for $\zeta \to \pm\infty$ for second approximation functions, derived in the same manner, are

$$v_{x22} \to \pm|\zeta|^{-(2+\varkappa)/2\varkappa} + O(|\zeta|^{-(2+5\varkappa)/2\varkappa}), \qquad v_{y22} \to |\zeta|^{-(2-\varkappa)/2\varkappa} + O(|\zeta|^{-(2+3\varkappa)/2\varkappa})$$
$$\rho_{22} \to \pm|\zeta|^{(2-\varkappa)/2\varkappa} + O(|\zeta|^{(2-5\varkappa)/2\varkappa}) \tag{3.4}$$
$$w_{22} \to \pm|\zeta|^{-(2+\varkappa)/2\varkappa} + O(|\zeta|^{-(2+5\varkappa)/2\varkappa}), \quad y_{22} \to |\zeta|^{-(2-\varkappa)/2\varkappa} + O(|\zeta|^{-(2+3\varkappa)/2\varkappa})$$

Let us substitute expansions (3.2) into the Navier-Stokes system of equations. Since the second of Eqs. (1.5) implies that $dp_{21}/d\xi = 0$ it is obvious that pressure across the hypersonic trail does not vary in the considered approximation. Allowing for boundary condition (3.3), we obtain $p_{21} = 1$. Taking this equality into account, after some simple transformations, we can write the system of first approximation equations thus

$$\frac{16\varkappa}{3(\varkappa^2-1)} h_0 \frac{1}{N_{Pr}} \frac{d^2 w_{21}}{d\zeta^2} + \zeta \frac{dw_{21}}{d\zeta} + \frac{1}{\varkappa} w_{21} = 0 \tag{3.5}$$
$$\frac{16\varkappa}{3(\varkappa^2-1)} h_0 \frac{d^2 v_{x21}}{d\zeta^2} + \zeta \frac{dv_{x21}}{d\zeta} + \frac{4\varkappa-3}{\varkappa} v_{x21} = \frac{4(\varkappa-1)}{\varkappa} w_{21}$$
$$\rho_{21} w_{21} = 1, \quad \rho_{21} \frac{dy_{21}}{d\zeta} = \frac{\varkappa-1}{\varkappa}, \quad \zeta \frac{dy_{21}}{d\zeta} - \frac{\varkappa+3}{\varkappa} y_{21} + \frac{4}{\varkappa} v_{y21} = 0$$

In this system the first equation is the key one, since after its integration it is possible to find solutions of the remaining equations. If $N_{Pr} = 1$ and $v_{x21} = w_{21}$, the results are considerably simplified [17]. For any arbitrary Prandtl number we substitute the independent variable

$$\zeta_1 = -\frac{3(\varkappa^2-1)}{32\varkappa} \frac{N_{Pr}}{h_0} \zeta^2 \tag{3.6}$$

the result of which is

$$\zeta_1 \frac{d^2 w_{21}}{d\zeta_1^2} + \left(\frac{1}{2} - \zeta_1\right) \frac{dw_{21}}{d\zeta_1} - \frac{1}{2\varkappa} w_{21} = 0$$

which is the canonical form of the so-called confluent hypergeometric equation [20]. Using the conventional notation for such functions, for their general solution we have

$$w_{21} = c_1 \Phi\left(\frac{1}{2\varkappa}, \frac{1}{2}; \zeta_1\right) + c_2(-\zeta_1)^{1/2} \Phi\left(\frac{\varkappa+1}{2\varkappa}, \frac{3}{2}; \zeta_1\right)$$

Turning now to formulas (3.1) and (3.6), we note that the first term in the right-hand side of the latter represents a solution symmetric about the streamline $\psi = 0$, while the second term provides an antisymmetric solution. The first approximation considered here defines the perturbation field in the trail associated only with the drag of the body in the stream. For such field the distribution of gas parameters in the upper and lower half-planes must, evidently, be symmetric. This implies that constant $c_2 = 0$, which is confirmed by the form of boundary conditions (3.3). For the determination of constant c_1 we use the asymptotic expansion of hypergeometric functions for $\zeta_1 \to -\infty$. Finally we obtain

$$w_{21} = \Gamma\left(\frac{\varkappa-1}{2\varkappa}\right)\Gamma^{-1}\left(\frac{1}{2}\right)\left[\frac{3(\varkappa^2-1)}{32\varkappa}\,\frac{N_{Pr}}{h_0}\right]^{1/2\varkappa}\Phi\left(\frac{1}{2\varkappa},\ \frac{1}{2};\ \zeta_1\right) \tag{3.7}$$

We pass to the second equation of system (3.5). Substituting the independent variable $\zeta_2 = \zeta_1 / N_{Pr}$, as the result we obtain the confluent hypergeometric equation

$$\zeta_2\,\frac{d^2 v_{x21}}{d\zeta_2^2} + \left(\frac{1}{2} - \zeta_2\right)\frac{dv_{x21}}{d\zeta_2} - \frac{4\varkappa-3}{2\varkappa}\,v_{x21} = -\frac{2(\varkappa-1)}{\varkappa}\,w_{21} \tag{3.8}$$

As the fundamental system of solutions of the homogeneous equation corresponding to (3.8) we select the integrals

$$v^{(1)}_{x21},\ v^{(2)}_{x21} = e^{\zeta_2}\left[\Gamma\left(\frac{1}{2}\right)\Gamma^{-1}\left(\frac{3-2\varkappa}{2\varkappa}\right)\Phi\left(-\frac{3(\varkappa-1)}{2\varkappa},\ \frac{1}{2};\ -\zeta_2\right) \pm \Gamma\left(-\frac{1}{2}\right)\Gamma^{-1}\left(-\frac{3(\varkappa-1)}{2\varkappa}\right)(-\zeta_2)^{1/2}\,\Phi\left(\frac{3-2\varkappa}{2\varkappa},\ \frac{3}{2};\ -\zeta_2\right)\right] \tag{3.9}$$

The plus and minus signs in the right-hand side of (3.9) relate to $v^{(1)}_{x21}$ and $v^{(2)}_{x21}$, respectively. It is convenient to consider integrals $v^{(1)}_{x21}$ and $v^{(2)}_{x21}$ in the plane of the complex variable $z = -\zeta_2 + i\omega$. In that plane the motion along the straight line $x = \mathrm{const}$ from the upper to the lower boundary of the vortex trail is accomplished by going around the positive real semiaxis in the direction shown in Fig. 3 by arrows. The linear combination of confluent hypergeometric functions contained in $v^{(1)}_{x21}$ may be written in the form of function [20]

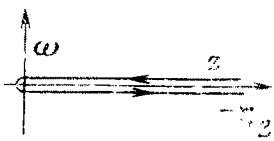

Fig. 3

$$\Psi\left(-\frac{3(\varkappa-1)}{2\varkappa},\ \frac{1}{2};\ z\right)$$

It will be seen that the linear combination of hypergeometric functions which define $v^{(2)}_{x21}$ reduces to the same Tricomi Ψ-function, if for its argument z we substitute z', where $|z'| = |z|$ and $\arg z' = \arg z - 2\pi$.

The Wronskian W_{21} of the selected fundamental system of solutions is specified by formula

$$\frac{1}{2}\Gamma\left(\frac{3-2\varkappa}{2\varkappa}\right)\Gamma\left(-\frac{3(\varkappa-1)}{2\varkappa}\right)\Gamma^{-2}\left(\frac{1}{2}\right)W_{21} = -e^{\zeta_2}z^{-1/2} = e^{\zeta_2}(z')^{-1/2}$$

and the particular solution v^*_{x21} of the nonhomogeneous equation (3.8) is

$$v^*_{x21} = \frac{2(\varkappa-1)}{\varkappa}\left(v^{(1)}_{x21}\int_0^{\zeta_2}\frac{w_{21}v^{(2)}_{x21}}{\zeta_2 W_{21}}\,d\zeta_2 - v^{(2)}_{x21}\int_0^{\zeta_2}\frac{w_{21}v^{(1)}_{x21}}{\zeta_2 W_{21}}\,d\zeta_2\right) \tag{3.10}$$

It is now obvious that

$$v_{x21} = c_3 v^{(1)}_{x21} + c_4 v^{(2)}_{x21} + v^*_{x21}$$

It remains to determine the arbitrary constants c_3 and c_4. If initially $\zeta \to +\infty$, then $\arg z = 0$ and $\arg z' = -2\pi$. To determine the principal terms of the expansion of integral $v^{(1)}_{x21}$ we use the asymptotics of the Ψ-function [20]. The asymptotic behavior of $v_{x21}^{(2)}$ is established by substituting asymptotic expansions for confluent hypergeometric functions into formula (3.9). These results can be used for proving that the principal term of the asymptotic expansion of the particular solution $\overset{*}{v}_{x21}$ corresponds exactly to conditions (3.3). Taking into account the order of correction terms appearing in these conditions, we obtain

$$c_4 = \frac{2(\varkappa - 1)}{\varkappa} \overline{\int_0^{-\infty}}^{\,*} \frac{w_{21} v_{x21}^{(1)}(z)}{\zeta_2 W_{21}(z)} d\zeta_2 \qquad (3.11)$$

where the small angled line over the improper divergent integral denotes its finite part in accordance with Hadamard's definition. The reasoning in the case of $\zeta \to -\infty$, $\arg z = 2\pi$ and $\arg z' = 0$ is similar. The behavior of integral $v_{x21}^{(1)}$ is determined by substituting asymptotic expansions for hypergeometric functions into formula (3.9), while the principal terms of the expansion of $v_{x21}^{(2)}$ are determined by the asymptotics of the Ψ-function. As the result we have

$$c_3 = -\frac{2(\varkappa - 1)}{\varkappa} \overline{\int_0^{-\infty}}^{\,*} \frac{w_{21} v_{x21}^{(2)}(z')}{\zeta_2 W_{21}(z')} d\zeta_2 \qquad (3.12)$$

Arguments of the complex variables z and z' in formulas (3.11) and (3.12) are zero. This with the properties of the Wronskian W_{21} taken into account implies that $c_3 = c_4$. Although both fundamental solutions $v_{x21}^{(1)}$ and $v_{x21}^{(2)}$ contain symmetric and antisymmetric parts, the particular solution $\overset{*}{v}_{x21}$ is symmetric. A simple test will show that for equal constants c_3 and c_4 function v_{x21} defines a velocity field which is symmetric about the streamline $\psi = 0$.

Let us pass to the fourth equation of system (3.5). Taking into consideration the order of correction terms in formulas (3.3) or the requirement for the symmetry of solution in the considered approximation, after its integration, we obtain

$$y_{21} = \frac{\varkappa - 1}{\varkappa} \Gamma\left(\frac{\varkappa - 1}{2\varkappa}\right) \Gamma^{-1}\left(\frac{1}{2}\right) \left[\frac{3(\varkappa^2 - 1)}{32\varkappa} \frac{N_{Pr}}{h_0}\right]^{1/2\varkappa} \zeta \Phi\left(\frac{1}{2\varkappa}, \frac{3}{2}; \zeta_1\right) \quad (3.13)$$

The fifth equation of system (3.5) can be converted into a finite relationship, hence

$$v_{y21} = \frac{\varkappa - 3}{4} y_{21} - \frac{\varkappa - 1}{4} \zeta w_{21}$$

where functions w_{21} and y_{21} are defined by equalities (3.7) and (3.13), respectively.

4. The second approximation. Let us investigate the effect of the application to the body of a lift force on the structure of the vortex trail in a hypersonic stream. First, let us determine the projection on the y-axis of the momentum component of gas transmitted through that part of the plane $x = \text{const}$ which contains the vortex trail (Fig. 1). By setting $x \to \infty$ and using expansion (3.2) we can readily show that the integral defining that component is insignificantly small. In other words, lift can be determined only by the parameters of the outer flow region. Calculations show that constant B in formulas (1.4) is equal 0.683.

Although the transfer of momentum of gas via the vortex trail can be neglected in the calculation of lift, the determination of associated perturbations of the velocity field requires that second terms in the right-hand sides of expansions (3.2) are taken into account. The latter satisfy the system of ordinary differential equations

$$\begin{gathered}
\frac{16\varkappa}{3(\varkappa^2-1)} h_0 \frac{1}{N_{Pr}} \frac{d^2 w_{22}}{d\zeta^2} + \zeta \frac{dw_{22}}{d\zeta} + \frac{\varkappa+2}{2\varkappa} w_{22} = 0 \\
\frac{16\varkappa}{3(\varkappa^2-1)} h_0 \frac{d^2 v_{x22}}{d\zeta^2} + \zeta \frac{dv_{x22}}{d\zeta} + \frac{9\varkappa-6}{2\varkappa} v_{x22} = \frac{4(\varkappa-1)}{\varkappa} w_{22} \\
\rho_{22} w_{21} - w_{22}\rho_{21} = 0, \qquad \frac{2\varkappa-1}{2-\varkappa}\rho_{21}\frac{dy_{22}}{d\zeta} + \rho_{22}\frac{dy_{21}}{d\eta} = 0 \\
\zeta \frac{dy_{22}}{d\zeta} - \frac{\varkappa+6}{2\varkappa} y_{22} + \frac{4}{\varkappa} v_{y22} = 0
\end{gathered} \tag{4.1}$$

whose structure is analogous to that of system (3.5). It follows from Eqs. (4.1) that in the considered approximation the hypothesis of plane sections is satisfied. In this case the key equation is again the first one which by the introduction of the independent variable by formula (3.6) reduces to the form

$$\zeta \frac{d^2 w_{22}}{d\zeta^2} + \left(\frac{1}{2} - \zeta\right)\frac{dw_{22}}{d\zeta} - \frac{\varkappa+2}{4\varkappa} w_{22} = 0$$

To satisfy boundary conditions (3.4) it is necessary to choose the antisymmetric solution

$$w_{22} = \Gamma\left(\frac{3\varkappa-2}{4\varkappa}\right)\Gamma^{-1}\left(\frac{3}{2}\right)\left[\frac{3(\varkappa^2-1)}{32\varkappa}\,\frac{N_{Pr}}{h_0}\right]^{(3\varkappa-2)/4\varkappa} \zeta\,\Phi\left(\frac{3\varkappa+2}{4\varkappa},\ \frac{3}{2};\ \zeta_1\right) \tag{4.2}$$

Perturbations of specific enthalpy resulting from this solution change their sign at intersection with the streamline $\psi = 0$. The introduction of the independent variable ζ_2 into the second equation of system (4.1) reduces it to

$$\zeta_2 \frac{d^2 v_{x22}}{d\zeta_2^2} + \left(\frac{1}{2} - \zeta_2\right)\frac{dv_{x22}}{d\zeta_2} - \frac{9\varkappa-6}{4\varkappa} v_{x22} = -\frac{2(\varkappa-1)}{\varkappa} w_{22} \tag{4.3}$$

As the fundamental system of solutions of the corresponding homogeneous equation (4.3) we take the integrals

$$\begin{gathered}
v^{(1)}_{x22},\ v^{(2)}_{x22} = e^{\zeta_2}\left[\Gamma\left(\frac{1}{2}\right)\Gamma^{-1}\left(-\frac{5\varkappa-6}{4\varkappa}\right)\Phi\left(-\frac{7\varkappa-6}{4\varkappa},\ \frac{1}{2};\ -\zeta_2\right) \pm \right. \\
\left. \Gamma\left(-\frac{1}{2}\right)\Gamma\left(-\frac{7\varkappa-6}{4\varkappa}\right)(-\zeta_2)^{1/2}\,\Phi\left(-\frac{5\varkappa-6}{4\varkappa},\ \frac{3}{2};\ -\zeta_2\right)\right]
\end{gathered} \tag{4.4}$$

where the plus and minus signs relate to $v^{(1)}_{x22}$ and $v^{(2)}_{x22}$, respectively. In the plane of the complex variable z the first of these, $v^{(1)}_{x22}$ is defined by the Tricomi function

$$\Psi\left(-\frac{7\varkappa-6}{4\varkappa},\ \frac{1}{2};\ z\right)$$

The second integral can be reduced to the same Ψ-function by the substitution of argument z' for z. The expression

$$\frac{1}{2}\Gamma\left(-\frac{5\varkappa-6}{4\varkappa}\right)\Gamma\left(-\frac{7\varkappa-6}{4\varkappa}\right)\Gamma^{-2}\left(\frac{1}{2}\right)W_{22} = -e^{\zeta_2}z^{-1/2} = e^{\zeta_2}(z')^{-1/2}$$

is valid for the Wronskian W_{22}.

Let the particular solution v^*_{x22} of the nonhomogeneous equation (4.3) be derived in accordance with the rule by which function v^*_{x21} is specified. It is then possible to apply to it equality (3.10) in which subscripts 22 are substituted for 21. For the general solution we have the formula

$$v_{x22} = c_5 v^{(1)}_{x22} + c_6 v^{(2)}_{x22} + v^*_{x22}$$

Arbitrary constants c_5 and c_6 are determined by boundary conditions (3.4) in which estimates of correction terms play an important part. Setting $\zeta \to +\infty$, we establish the principal terms of expansion $v_{x22}^{(1)}$ on the basis of the asymptotics of the Tricomi Ψ-function, and use formula (4.4) for computing the integral $v_{x22}^{(2)}$. For $\zeta \to -\infty$ the behavior of $v_{x22}^{(1)}$ is determined by substituting asymptotics of confluent hypergeometric functions into formula (4.4), while the principal terms of expansion of integral $v_{x22}^{(2)}$ are determined by using its expression in terms of the Ψ-function.

We note that every point of the contour shown in Fig. 3 satisfies the equality

$$\zeta = \left[\frac{32\varkappa}{3(\varkappa^2-1)} h_0 z\right]^{1/2} = -\left[\frac{32\varkappa}{3(\varkappa^2-1)} h_0 z'\right]^{1/2}$$

Substituting it into the right-hand side of formula (4.2), we finally obtain

$$c_5 = -\frac{2(\varkappa-1)}{\varkappa}\int_0^{-\infty *} \frac{w_{22}(z')\, v_{x22}^{(2)}(z')}{\zeta_2 W_{22}(z')}\, d\zeta_2$$

$$c_6 = \frac{2(\varkappa-1)}{\varkappa}\int_0^{-\infty *} \frac{w_{22}(z)\, v_{x22}^{(1)}(z)}{\zeta_2 W_{22}(z)}\, d\zeta_2$$

where the arguments of the two complex variables z and z' are zero and, consequently, $c_5 = -c_6$. It can be shown that under this condition the velocity field perturbations associated with function v_{x22} are antisymmetric with respect to the streamline $\psi = 0$.

With allowance for boundary conditions (3.4) integration of the fourth equation of system (4.1) yields

$$y_{22} = \Gamma\left(\frac{3\varkappa-2}{4\varkappa}\right)\Gamma^{-1}\left(\frac{1}{2}\right)\left[\frac{3(\varkappa^2-1)}{32\varkappa}\frac{N_{Pr}}{h_0}\right]^{(2-\varkappa)/4\varkappa}\Phi\left(\frac{2-\varkappa}{4\varkappa}, \frac{1}{2}; \zeta_1\right) \quad (4.5)$$

The fifth equation of that system yields

$$v_{y22} = \frac{2-\varkappa}{8}\zeta w_{22} + \frac{\varkappa+6}{8} y_{22}$$

where functions w_{22} and y_{22} are defined by equalities (4.2) amd (4.5).

Under the effect of lift the zero streamline is no longer the axis of symmetry of the flow, whose equation, derived from the last of expansions (3.2), is

$$y_0 = -\frac{4(\varkappa-1)}{(\varkappa+1)(2-\varkappa)}\Gamma\left(\frac{3\varkappa-2}{4\varkappa}\right)\Gamma^{-1}\left(\frac{1}{2}\right)\left[\frac{3(\varkappa^2-1)}{32\varkappa}\frac{N_{Pr}}{h_0}\right]^{(2-\varkappa)/4\varkappa} \times \quad (4.6)$$
$$b^{2/3} b_{1/2} h_0^{-1/\varkappa} x^{(\varkappa+6)/12\varkappa} + \ldots$$

This shows that the displacement y_0 of the zero streamline from its initial position infinitely increases with distance from the body. Comparison of formulas (1.4) and (4.6) shows that the zero streamline deflects in the direction of lift F_y. This is not a trivial conclusion and is explained by the previously noted possibility of neglecting the transfer of momentum of gas through that part of the reference plane $x = \text{const}$ which comprises the vortex trail.

5. Particular cases. The value $\varkappa = 3/2$ is particular for the first approximation solution which describes the trail flow. One of the linearly independent integrals of the related homogeneous equation (3. 8) may then be represented by the equality

$$v_{x21}^{(0)} = e^{\zeta_2} \, (-\zeta_2)^{1/2}$$

Owing to exponential attenuation with $\zeta \to \pm\infty$ this equality is, obviously, the intrinsic solution of the problem. A similar situation occurs when second approximation formulas are analyzed for $\varkappa = {}^6/_5$. In that case we take the above eigenfunction $v_{x22}^{(0)} = v_{x21}^{(0)}$ as one of the fundamental integrals of Eq. (4. 3) in which the right-hand side is equated to zero. Thus for $\varkappa = {}^3/_2$ or ${}^6/_5$ the velocity field within the vortex trail is not uniquely defined by the method of matching outer and inner asymptotic expansions.

We note in conclusion that the value of exponent $m = {}^1/_2$ which is determined by the requirement for a finite lift, may in a certain sense be considered to be an eigenvalue. For instance, let a flow be specified which in the outer region satisfies expansions (1. 3). It is required to extend this flow to the neighborhood of the streamline $\psi = 0$. It appears that at least in the range $0 < m < 1$ the velocity field pertains to the free stream in the vortex trail only for $m = {}^1/_2$, while for any other m it is necessary to introduce a half-body which extends to infinity. The results of the investigation of hypersonic trail, presented above, are essentially based on the fact that in accordance with the asymptotic formulas (2. 5) the pressure perturbation $p_{12} \to 0$ when $|\eta| \to 0$. For $m \neq {}^1/_2$ the asymptotic expansion of function p_{12} commenses with a certain constant whose magnitude for $\eta \to +0$ is not the same as that for $\eta \to -0$.

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IRREGULAR INTERACTION OF WEAK SHOCK WAVES OF DIFFERENT INTENSITIES

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The problem of irregular interaction of weak shock waves, which occurs in the analysis of interpenetration of two waves of different intensities at small interaction angle [1, 2], is considered. It is not possible to solve this problem in linear configuration when the region adjacent to the Mach wave front shrinks to a point, which results in it becoming a nonlinear problem. Behavior of the solution throughout the interaction region is analyzed by the method of matching asymptotic expansions [3, 4]. The external problem is solved in linear formulation. A boundary value problem for the system of nonlinear equations of short waves [5], which takes into account the linking of its solution with the linear external problem and with solutions in the neighborhood of reflected fronts at the inner region boundary, is formulated for the inner region in the neighborhood of the Mach wave front. The effect of the initial state parameters on the pattern of flow is investigated and an approximate solution of the problem is derived.

1. Let us consider the interaction of two plane shock waves in a stationary perfect polytropic gas running off a wedge of angle α (Fig. 1, a). Let the waves meet at instant of time $t = 0$ at point O and begin to interact. We select the system of coordinates so that the Ox-axis lies along the wedge axis of symmetry. For weak shock waves of